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Calculation of the macro absorption factor for a cylindrical specimen irradiated with a fine beam.

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In the investigation of diffraction effects in metallic and fibrous substances it is often convenient to use a cylindrical specimen whose diameter, $2R$, is greater than the width, $2r$, of the perpendicularly incident beam.

Bradley (1935) and Taylor & Sinclair (1945) have considered the case $2r > 2R$, but details of the present system do not appear to have been reported.

Fig. 1 shows the experimental arrangement and the

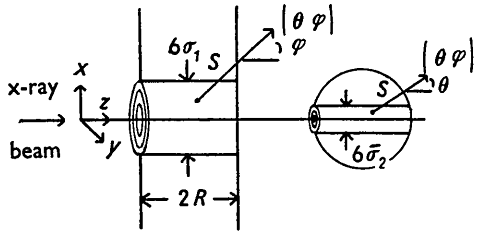

Fig. 1. The experimental arrangement, showing the incident beam intensity contours represented by a series of homothetic ellipses, φ the latitude and θ the longitude corresponding to a reflecting plane, S the radius vector from a volume element $dx\,dy\,dz$ in the direction θ, φ bounded by the cylinder boundary, x, y, z the spatial co-ordinates.

parameters involved. The integrated intensity from a plane (hkl) in the direction θ, φ is given by:

$$I_{(\theta,\varphi)} = \iiint_V \psi(x, y, z) \exp\,[-f(x, y, z)]dxdydz\,,$$

where $\psi(x, y, z)$ represents the intensity of the incident radiation, and $f(x, y, z)$ is the absorption path length multiplied by the linear absorption coefficient μ, for each volume element $dxdydz$ at the point x, y, z. Since only the integrated intensity is of interest here it is convenient to neglect beam divergence. The intensity of the incident beam may be represented at a given value of z by the function:

$$\psi(x, y)_z = \frac{1}{2\pi\sigma_1\sigma_2} \exp - \left(\frac{x^2}{2\sigma_1^2} + \frac{y^2}{2\sigma_2^2}\right)$$

and put into a form amenable to calculation by considering at intervals along z families of homothetic ellipses

$$x^2/\sigma_1^2 + y^2/\sigma_2^2 = c^2$$

which are the loci of equal intensity. From experimental observations and collimation geometry it is reasonable to put 6σ equal to the width of the collimator, which, in the case of a circular beam, means $3\sigma_1 = 3\sigma_2 = 3\sigma = r$. When using a rectangular incident beam with slit collimation ($\sigma_1 \neq \sigma_2$) a similar relationship applies and the ellipticity of the homothetic ellipses is then equal to σ_1/σ_2.

If the case is considered where $R/r \geq 5$, then the absorption path length associated with each volume element is given fairly accurately by $Z+S$, where S is the positive real root of

$$S^2 \cos^2 \varphi + 2S(Z-R) \cos \varphi \cos \theta - 2RZ + Z^2 = 0\,.$$

In practice it is convenient to sum volume elements of arc-length $\pi/5$ of six homothetic ellipses at spacings of $R/5$ along the Z axis. This is a lengthy operation and graphical integration is sufficiently accurate for most cases. If $r \ll R$ there are two points of interest. First, if φ and θ are small the simple case of linear absorption arises, and secondly if $\theta = 0$ and $0 < \varphi < \frac{1}{2}\pi$ we have essentially transmission by a block sample of infinite length. This has been treated by Gingrich (1943) and yields the explicit relationship

$$\frac{I_\varphi}{I_0} = \frac{1-\exp\,(-2\mu R\,(\sec \varphi - 1))}{2\mu R\,(\sec \varphi - 1)}\,.$$

Fig. 2 compares the intensities for meridional and equatorial directions of a sample irradiated with a beam

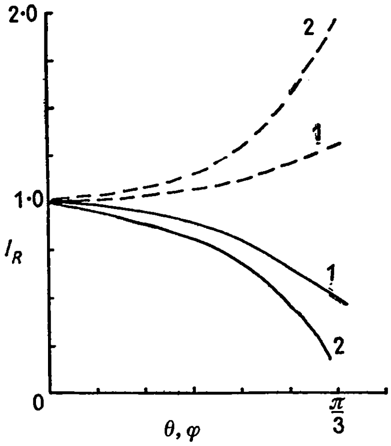

Fig. 2. Change in intensity (from $\varphi = \theta = 0$) for the case of meridional and equatorial reflections with $\mu R = 1$ and 2 over the range $0 < \varphi, \theta < \frac{1}{3}\pi$. Broken line: equatorial; full line: meridional.

of circular cross-section with $R/r = 5$, for the cases $\mu R = 1$ and $\mu R = 2$. It will be seen that the effect is important, but the number of parameters involved makes it necessary to restrict calculations to specific cases. It will be noticed that a locus can be constructed giving points where $I_{(\theta,\varphi)} = I_{(\theta=\varphi=0)}$ and no corrections are needed. The intensity distribution can also be constructed from the data but in order to obtain accurate profile sections beam divergence must be considered. Tilting of the specimen may also alter the integrated and distributed intensity, but can be dealt with by a development of the method outlined.

References

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